Nilpotent Symplectic alternating algebras

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Overview

1: Introduction.

2: Some General theory.

3: Presentations of Nilpotent SAAs.

4: Algebras of Maximal class.

5: Applying theory to practice.

6: Classification of Algebras.

1. Introduction

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SAA's originate in a study of powerful 2-Engel groups. These are structures that also seem to be interesting in their own right with many beautiful properties and this is how we approach them in this work.

Definition. Let F be a field. A SAA over F is a triple $(V, (\ ,\ ), \cdot)$ where V is a symplectic vector space over F with respect to a non-degenerate alternating form $(\ ,\ )$ and $\cdot$ is a alternating bilinear and binary operation on V such that

$$(u \cdot v, w) = (v \cdot w, u) \quad (*)$$

for all $u, v, w \in V$.

Remark. (*) is equivalent to $(u \cdot x, v) = (u, v \cdot x)$

Presentation. Let $(u_1, \ldots, u_{2n})$ be any basis for L. The structure of L is determined by the non-zero triple values

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$$L: \quad (x_1 y_1, y_2) = 1.$$

General Properties (I) .

Let L be a SAA of dim $2n$.

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(5) Any abelian-by-nilpotent algebra is nilpotent. (Tota,Tortora,Traustason, 2012)

Definition. A symplectic alternating algebra L is ***nilpotent*** if there exists an ascending chain of ideals $I_0, I_1, \cdots, I_n$ such that

$$\{0\} = I_0 \leqslant I_1 \leqslant \cdots \leqslant I_n = L$$

and $I_s L \leqslant L_{s-1}$ for $s = 1, 2, \cdots, n$. The smallest possible n is then called the nilpotence class of L.

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$$\text{rank}(L) = \dim Z(L).$$

Proof. We have $\text{rank}(L) = \dim L - \dim L^2 = \dim (L^2)^{\perp} = \dim Z(L)$.

In particular there is no nilpotent SAA where $Z(L)$ is 1-dimensional.

Some Further General Properties (II)

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Lemma . Let L be a nilpotent SAA and I be an ideal of dimension 2. Then $I \leqslant Z(L)$. Equivalently every ideal of co-dimention 2 must contain L^2.

Proof. As any 2-dimensional nilpotent alternating algebras is abelian .

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Proposition . We have that $\dim L^i \neq 2$ for $2 \leqslant i \leqslant 4$. Equivalently $Z_i(L)$ is not of co-dimension 2 if $1 \leqslant i \leqslant 3$.

Example. Let L be the nilpotent SAA with presentation

$$\mathcal{P}: \quad (x_2y_3, y_4) = 1, (x_1y_2, y_3) = 1, (y_1y_2, y_4) = 1.$$

Then $\dim L^5 = 2$.

Theorem 2.

Let L be a nilpotent SAA of dim $\geqslant 2$. There exist an ascending chain of isotropic ideals

$$\{0\} = I_0 < I_1 < I_2 < \cdots < I_{n-1} < I_n$$

such that dim $I_k = k$, where $0 \leqslant k \leqslant n$. Furthermore for $2n \geqslant 6$, the ascending chain

$$\{0\} = I_0 < I_2 < I_3 < \cdots < I_{n-1} < I_{n-1}^{\perp} < I_{n-2}^{\perp} < \cdots < I_2^{\perp} < I_0^{\perp} = L$$

is a central chain s.t. $I_{n-1}^{\perp}$ is Abelian. In particular L is nilpotent of class at most $2n-3$.

Presentation. We can pick a standard basis $(x_1, y_1, \cdots, x_n, y_n)$

$$I_0 = \{0\}, I_1 = Fx_n, I_2 = I_1 + Fx_{n-1}, \cdots, I_n = I_{n-1} + Fx_1,$$

$I_{n-1}^{\perp} = I_n + Fy_1$ abelian $, I_{n-2}^{\perp} = I_{n-1}^{\perp} + Fy_2, \cdots, I_0^{\perp} = L = I_1^{\perp} + Fy_n$

$$\begin{array}{rcc l}
 & x_n & y_n & \\
I_{n-1} & \vdots & \vdots & \\
 & x_2 & y_2 & \\
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$$\begin{array}{ccc} & \boxed{\begin{array}{c} x_n \\ \vdots \\ x_2 \end{array}} & \begin{array}{c} y_n \\ \vdots \\ y_2 \end{array} \\ I_{n-1} & & \\ & \boxed{x_1 \quad y_1} & I_{n-1}^{\perp} \;\; \textit{Abelian} \end{array}$$

At most the non-zero triples are

$(x_i y_j, y_k) = \alpha_{ijk}$ and $(y_i y_j, y_k) = \beta ijk \quad 1 \leqslant i < j < k \leqslant n \quad (*)$

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I_{n-1}: $x_n \quad y_n$

$\vdots \quad \vdots$

$x_2 \quad y_2$

$x_1 \quad y_1$ $I_{n-1}^{\perp}$ *Abelian*

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$$(x_i y_j, y_k) = \alpha_{ijk} \text{ and } (y_i y_j, y_k) = \beta ijk \quad 1 \leqslant i < j < k \leqslant n \quad (*)$$

Conversely any such presentation (*) gives us a nilpotent SAA with ascending chain $I_1 < I_2 < \cdots < I_n$ where dim $I_r = r$ for $r = 1, \ldots, n$ just by letting

$$I_r = Fx_n + Fx_{n-1} + \cdots + Fx_{n+1-r}.$$

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$$Z_3(L) = I_4 \; \boxed{x_{n-3}} \; \boxed{y_{n-3}} \; {I_3}^{\perp} = Z_{2n-5}(L)$$

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Furthermore $Z_0(L), Z_1(L), \cdots, Z_{2n-3}(L)$ are the unique ideals of L of dimensions $0, 2, 3, \cdots, n-1, n+1, n+2, \cdots, 2n-2, 2n$.

Theorem 4.

Let L be a nilpotent SAA of dimension $2n \geqslant 10$ that is of maximal class. L has a chain of characteristic ideals

$$\{0\} = I_0 < I_1 < \cdots < I_n < I_{n-1}^{\perp} < \cdots < I_1^{\perp} < I_0^{\perp} = L$$

where for $0 \leqslant k \leqslant n$, I_k is isotropic of dimension k.

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$$\begin{array}{rcl}
Z(L) = I_2 & \boxed{\begin{array}{c} x_n \\ x_{n-1} \end{array}} \; \begin{array}{c} y_n \\ y_{n-1} \end{array} & \\
Z_2(L) = I_3 & \boxed{x_{n-2}} \; \boxed{y_{n-2}} & {I_2}^{\perp} = Z_{2n-4}(L) \\
Z_3(L) = I_4 & \boxed{x_{n-3}} \; \boxed{y_{n-3}} & {I_3}^{\perp} = Z_{2n-5}(L) \\
 & \vdots \qquad \vdots & \\
Z_{n-2}(L) = I_{n-1} & \boxed{x_2} \; \boxed{y_2} & {I_{n-2}}^{\perp} = Z_n(L) \\
 & \boxed{x_1 \qquad y_1} & {I_{n-1}}^{\perp} = Z_{n-1}(L)
\end{array}$$

Theorem 5.

Let L be a nilpotent SAA of dimension $2n \geqslant 8$ given by some nilpotent presentation $\mathcal{P}$. The algebra is of maximal class if and only if $x_k y_{k+1} \neq 0$ for all $k = 2, \cdots, n-2$ and $x_1 y_2, y_1 y_2$ are linearly independent.

$$Z(L) = I_2 \quad \boxed{\begin{matrix} x_n \\ x_{n-1} \end{matrix}} \quad \begin{matrix} y_n \\ y_{n-1} \end{matrix}$$

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5. Algebras of $dim\ 8$
Application: A maximal class algebra.

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$$\mathcal{P}_8{}^{(2,3)}(r): \quad (x_2y_3, y_4) = r, \ (x_1y_2, y_4) = 1, \ (y_1y_2, y_3) = 1.$$

$$\mathcal{P}_8{}^{(2,2)}(r) \sim \mathcal{P}_8{}^{(2,3)}(s) \Leftrightarrow s/r \in (\mathbb{F}^*)^3.$$

When $Z(L)$ is non-isotropic. Let $I = Fx_1 + Fy_1$. Then $L = I \oplus I^{\perp}$ as SAA's, where $dim\ I^{\perp} = 8$. So,

$$L = (Fx_1 + Fy_1) \oplus \begin{pmatrix} x_5 & y_5 \\ x_4 & y_4 \\ x_3 & y_3 \\ x_2 & y_2 \end{pmatrix}$$

Hence the problem here reduces to finding nilpotent SAA's of dimension 10 with isotropic center.

The study reveals there are 22 such algebras when the field is algebraically closed. However, Over $GF\ (3)$ there are 25 algebras.

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- I would like to thank my supervisor Professor Gunnar Traustason.
- Many thanks to the European Mathematical Society (EMS-CDC) for their generous support.

T H A N K   Y O U