

On the Verification of ATLAS Information

Thomas Breuer

Lehrstuhl D für Mathematik, RWTH Aachen University, Germany

Groups St Andrews, Birmingham, August 11, 2017

I will report on details and side-effects of the recent verification of most of the character tables in the ATLAS of Finite Groups.

This is joint work with Gunter Malle and Eamonn O'Brien.

(The paper has now appeared in
“Finite Simple Groups: Thirty Years of the Atlas and Beyond”,
vol. 694 of Contemporary Mathematics, 2017)

In November 2015 ...

... Gunter Malle and Eamonn O'Brien came up with the idea to verify the character tables in the ATLAS of Finite Groups, addressing J.-P. Serre's question how reliable the ATLAS data are.

Verification Idea, Part I:

For a given ATLAS character table,

- Take a representation of the underlying group.

Many such representations are available in Rob Wilson's ATLAS of Group Representations.

- Compute the character table from that representation.

Try MAGMA's standard method (Unger's method) on a sufficiently large computer.

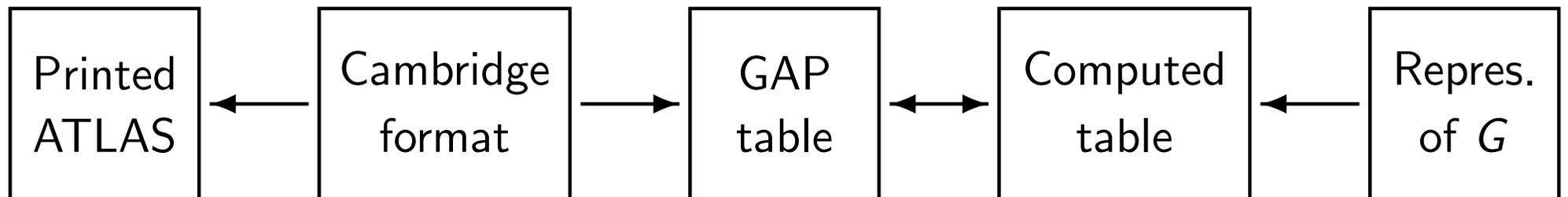
- Compare the result with the ATLAS table.

Check permutation equivalence –easy for two character tables in GAP.

Verification Idea, Part II:

Apply part I to all ATLAS tables.

So first we have to define what we regard as ATLAS tables.



Verification Idea, Setup:

We assume that the “Cambridge format” files correspond to the printed ATLAS (modulo the known improvements).

We assume that the GAP tables correspond to these files.

We check the GAP tables against the computed tables.

What are the Relevant Groups?

Bicyclic extensions of nonabelian simple groups,
since the ATLAS claims on p. xxii:

ATLAS tables give information only about bicyclic extensions.

$$\begin{array}{ccc} G & \hookrightarrow & G.A & G \text{ simple} \\ \uparrow & & \uparrow & G.A \leq \text{Aut}(G), A \text{ cyclic} \\ M.G & \hookrightarrow & M.G.A & M.G \text{ perfect, } M \text{ cyclic, central} \end{array}$$

What are the Relevant Groups?

- 88 simple groups:
Take the list from the “Contents” page,
omit the ones whose table is not shown.
- 102 non-simple almost simple groups:
Count classes of cyclic subgroups of $Out(G)$.
- 80 non-simple perfect central extensions:
Count $Out(G)$ -orbits of cyclic factors of $Mult(G)$.
- 125 proper bicyclic extensions, up to isoclinism:
Count the relevant squares in ATLAS Maps.
- 4 “broken box” cases:
 $4.A_6.2_3$, $12.A_6.2_3$, $4.L_2(25).2_3$, and $9.U_3(8).3_3$.
(The GAP tables were not computed from ATLAS information.)
- “Assign an ATLAS name to each of the chosen representations.”

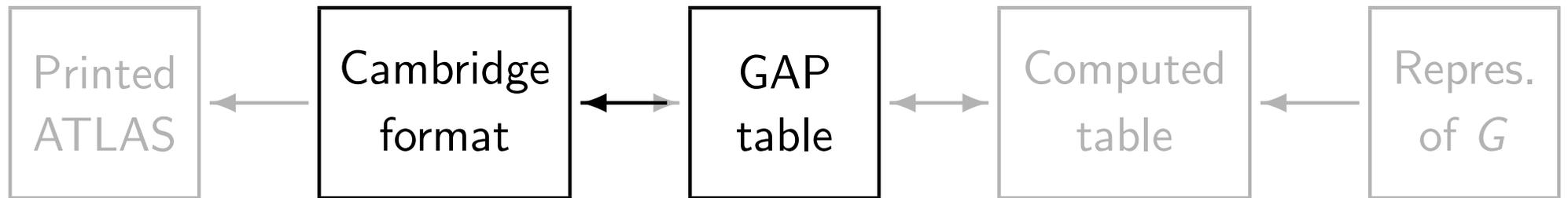
Verification Results

- Computing the character table with MAGMA worked in all except the following cases: J_4 , $2.^2E_6(2)$, $2.^2E_6(2).2$, B , $2.B$, and M .
- No ATLAS error was found this way.
- We found an error in MAGMA's 'print' routine.
- We found an error in the 2-power map of GAP's character table of $2.F_4(2).2$.
- The tables of $12_1.U_4(3).2'_2$ and $12_2.U_4(3).2'_3$ had been missing up to now from GAP's library of character tables.
- The Improvements list contained an irritating remark about $G = U_3(8)$:
"There is a unique group $3.G.6$, and $3.G.6'$, $3.G.6''$ do not contain $3.G.3_1$ but its isoclines."
- We have now a representation for almost all ATLAS groups.

Verification Results, the Remaining Six Groups

- $G = J_4$:
Janko's paper describes class representatives, centralizer orders, and power maps.
Assume the character table of $2^{11} : M_{24} \leq G$, determine the irreducible characters of G character-theoretically.
- $G = 2.^2E_6(2)$:
Use the known character tables of ${}^2E_6(2)$ and of subgroups $2.F_4(2)$, $2.Fi_{22}$, $2.U_6(2)$, $O_{10}^-(2)$, determine conjugacy classes and irreducible characters of G .
- $G = 2.^2E_6(2).2$:
Do the same, using also the table of $2.^2E_6(2)$.
- $2.B$: **If we assume** that the table of B is correct, induce from $2.^2E_6(2).2$ and $2 \times Th$.
- B : (Wilson, Magaard; July 2017)
Use explicit computations for establishing class representatives, use the tables of $2.^2E_6(2).2$, $2^{1+22}.Co_2$, Fi_{23} , $HN.2$.

Another Verification Step



Create “Cambridge format” from GAP format, and then compare with the source files.

Did we find new errors?

Cambridge Format \rightarrow GAP Format

New errors/inconsistencies:

- duplicate class names for $U_3(4)$, $U_4(3)$
- wrong element orders for $4.L_2(25).2_3$
- unfortunate choice of preimage classes for $4.U_4(3).2_1$
- swapped fusion markers for $L_4(3)$

How Many Errors are in the ATLAS?

164 entries are marked with * * *.

Tagged version of the errors in the ATLAS Improvements lists:

CH:	24	character values
P:	37	power maps
FI:	9	fusions and indicators
I:	14	Introduction, Bibliography, List showing $Mult(G)$ and $Out(G)$
MP:	3	maps
MX:	51	description of maximal subgroups
G:	26	other information about the group

Explicit Data Available in the Web

- Input data for the verification and overview of results:
<http://www.math.rwth-aachen.de/~Thomas.Breuer/atlasverify/bibl>
(group names, links to representations, runtimes)
- Improvements to the ATLAS (Part II) and to the ABC:
<http://brauer.maths.qmul.ac.uk/Atlas/info/fullatlasmods.html>,
<http://www.math.rwth-aachen.de/~MOC/ABCerr.html>
- Preprint of the paper:
<https://arxiv.org/pdf/1603.08650>
- Documentation of the character theoretic constructions:
<https://arxiv.org/pdf/1604.00754>

Thank you for your attention.