# Groups St Andrews 2013

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# Contents

**Logically and algebraically homogeneous groups**

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In our work we study algebraic structures within the frames of logical geometry.

One of the working tools in universal algebra and model theory is homogeneity. In this context, we consider the homogeneity property for algebras, in particular, for groups.

An algebra $H$ is *algebraically homogeneous* if every isomorphism between two of its finitely generated subalgebras can be extended up to an automorphism of $H$.

The definition of logical homogeneity is based on the same idea as the notion of algebraic homogeneity and will be given in the talk. Being close by their nature, these notions have also a lot of distinction. Every algebraically homogeneous algebra is logically homogeneous. The inverse statement is not true: two finitely generated free abelian groups are logically, but not algebraically homogeneous.

The logical homogeneity can be described using the model-theoretical notion of *a type*. Along with the model-theoretical types we consider *logically-geometrical types*. The last ones provide the bridge between logic and geometry.

The talk is focused on logically homogeneous groups. We formulate some results and discuss open problems.

**New progress on factorized groups and subgroup permutability**

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The study of products of groups whose factors are linked by certain permutability conditions has been the subject of fruitful investigations by a good number of authors. A particular starting point was the interest in providing criteria for products of supersoluble groups to be supersoluble. We take further previous research on total and mutual permutability by considering significant weaker permutability hypotheses. The aim of this talk is to report about new progress on structural properties of factorized groups within the considered topic. As a consequence, we discuss new attainments in the framework of formation theory.

**Rational conjugacy of torsion units in integral group rings of non-solvable groups**

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We present a new method to examine rational conjugacy of torsion units in integral group rings. The approach involves modular and integral representation theory and is especially interesting when combined with the standard HeLP-method. Let $\mathrm{V}(\mathbf{Z}G)$ denote the group of augmentation one units of the integral group ring $\mathbf{Z}G$. We prove the following

**Theorem.** If G is PSL(2,19) or PSL(2,23), then all torsion units of $\mathrm{V}(\mathbf{Z}G)$ are conjugate within the corresponding rational group ring to an element of the group. Furthermore, there are no units of order 6 in $\mathrm{V}(\mathbf{Z}G)$ provided G is isomorphic to $M_{10}$, the Mathieu group of degree 10, or PGL(2,9).

The first part shows that the long-standing (first) Zassenhaus conjecture holds for the groups mentioned. The second part completes the proof of a theorem of W. Kimmerle and A. Konovalov stating that the prime graph of the group G coincides with that one of the group of augmentation one units $\mathrm{V}(\mathbf{Z}G)$, provided the order of G is divisible by at most 3 different primes.

**On Clifford-Fischer Theory**

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Bernd Fischer presented a powerful and interesting technique, known as Clifford-Fischer theory, for calculating the character tables of group extensions. This technique derives its fundamentals from the Clifford theory. The present article surveys the developments of Clifford-Fischer theory applied to group extensions (split and non-split) and in particular we focus on the contributions of the authors to this domain.

**Thompson-like groups acting on Julia sets**
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We describe a family of groups acting on Julia sets for certain quadratic polynomials. These groups are similar to the Thompson groups $F$, $T$, and $V$, as well as the diagram groups of Victor Guba and Mark Sapir. We will discuss the action of these groups on certain cubical complexes, and the resulting finiteness properties.

**A generalisation on the solvability of finite groups with three class sizes for normal subgroups**
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A renowned theorem by N. Itô asserts that groups having exactly three class sizes are solvable. The proof of this result was improved by A. Camina and by J. Rebmann and has virtually remained unchanged since the 70's. In this talk, we present a sketch of the proof of the following generalisation of Itôs theorem for normal subgroups: If $G$ is a finite group, then every normal subgroup of $G$ that has exactly three $G$-conjugacy class sizes is solvable. Our approach, which uses the Classification of the Finite Simple Groups, has the advantage of enabling to argue by induction.

**Shift Dynamics and Asphericity for Cyclically Presented Groups**
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For group presentations with cyclic symmetry, there is a connection between asphericity of the (two-complex modeled on the) presentation and the dynamics of the cyclic shift automorphism for the group defined by the presentation. Specifically, if the presentation is combinatorially aspherical and orientable, then the shift and its powers are fixed-point-free on the non-identity elements of the group. For cyclic presentations with positive relators of length three, as studied by M. Edjvet and G. Williams, the converse holds and moreover the shift itself has a non-identity fixed point if and only if the group is finite.

**Approximate groups.**

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Approximate groups are finite subsets of an ambient group $G$, which are almost closed under multiplication. The notion was introduced by T. Tao in 2005 and has attracted a lot of attention in recent years, in part because of its use in constructing new families of expander graphs. My lectures will attempt to describe these recent developments.

**Orbit coherence in permutation groups**

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For a permutation $g$ of a set $X$, let $p(g)$ be the partition of $X$ given by the orbits of $g$. For a permutation group $G$ on $X$, let $p(G)$ be the set of partitions $p(g)$ for $g$ in $G$. The set of all partitions of $X$ forms a complete lattice under the refinement order, and the subset $p(G)$ inherits an order structure. I shall talk about some recent work (joint with Mark Wildon) on permutation groups $G$ for which $p(G)$ is an upper- or lower-subsemilattice.

**Anisimov's Theorem for Inverse Semigroups**

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An inverse semigroup is a semigroup in which every element has a unique inverse ($x$ is an inverse of $y$ if $xyx = x$ and $yxy = y$).

The seminal result in the study of word problems of groups considered as formal languages was Anisimov's Theorem (1971), which states that a group has regular word problem if and only if it is finite. I will present two quite different ways of generalising the group word problem to inverse semigroups, both of which give rise to versions of Anisimov's theorem, one due to myself and the other to Mark Kambites.

**Schur sigma-groups**

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Schur sigma-groups are a type of pro-$p$ group with balanced presentation and distinguished automorphism sigma, that were first identified by Koch and Venkov in 1975. They arise as Galois groups in a natural way. I'll discuss some recent results and questions concerning these groups.

**Permutation groups and transformation semigroups**

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I will talk about the way in which our knowledge of groups can help us study semigroups; in particular, new results about permutation groups which were motivated by applications to transformation semigroups.

In the first part of the talk, I discuss some recent results on the semigroups $\langle a, G\rangle \setminus G$ and $\langle g^{-1}ag : g \in G\rangle$, where $G$ is a permutation group and $a$ a map which is not a permutation. Typical questions are: when are these semigroups equal? When, for given groups $G$ and $H$, do they coincide? When do they have nice properties such as regularity or idempotent-generation? These lead to questions about new concepts in permutation group theory such as $\lambda$-homogeneity (where $\lambda$ is a partition), $(k, l)$-homogeneity where $k < l$, and the $k$-universal transversal property.

The second part is a brief report on the synchronization project, the attempt to answer the question: for which permutation groups $G$ is it true that $\langle a, G\rangle$ contains a map of rank 1 for any non-permutation $a$? The obstruction to this property turns out to be endomorphisms of very special graphs; but these lead to hard geometric and combinatorial problems about permutation groups.

**A construction for the outer automorphism of $S_6$**

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In joint work with Neil Gillespie and Cheryl Praeger on the construction of neighbour transitive codes from complex Hadamard matrices, we came across what appears to be a new construction for the outer automorphism of the symmetric group $S_6$.

In this talk I will describe the construction, given by two representations of $3.S_6$ which are intertwined by a complex Hadamard matrix.

**A non-embedding result for R. Thompson's group V**

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Thompson's group $V$ was first defined in 1965. It can be interpreted as a particular subgroup of the automorphism group of the Cantor set. We use some dynamical properties of the action of an element of $V$ on the Cantor Set of to show that $\mathbb{Z} \wr \mathbb{Z}^2$ does not embed into Thompson's group $V$. This result adds to the limited number of structure theorems for $V$. Part of the interest in a dynamical approach stems from the historical difficulty of purely algebraic techniques to obtain structure results about $V$.

**Finite groups acting on groups**

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Let $G$ be a finite group and let $A = Aut(G)$. We give a "combinatorial" characterization of the situation when $A$ is abelian.

We also show that, when $G$ is nilpotent of class $n$ and $\alpha \in A$, the number $|G : C_G(\alpha)|$ is a product of the orders of $n$ precisely defined abelian groups.

**The 6-transposition Coxeter groups $G^{(m,n,p)}$**

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We give a complete classification of all groups having the following property (6).

Property (6): A group $G$ has property (6) if it satisfies the following two conditions:
(i) $G$ is generated by three involutions $\{a, b, c\}$ two of which commute, say $ab = ba$;
(ii) We let $T$ be the union of the conjugacy classes of $a$, $b$, $ab$, and $c$. For any two involutions $t$ and $s$ in $T$, the product $ts$ has order at most 6. We say that $T$ is '6-transposition'.

In this talk I will give a complete classification of the groups having property (6). I will then explain how the motivation for this classification originated in Majorana Theory, which is an axiomatisation of some of the properties of the Monster algebra, as introduced by A. A. Ivanov. To this end, Majorana Theory and Majorana representations will be very briefly presented.

**Recent advances in computing with infinite linear groups**

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In the talk we will discuss recent progress in computing with finitely generated linear groups over infinite fields. Main consideration will be given to linear groups of finite (Prüfer) rank. For a finitely generated linear group $G$ over a number field we develop algorithms testing whether $G$ is of finite rank, and if so, we compute the torsion free rank of $G$. This yields in turn an algorithm to decide whether a finitely generated subgroup of $G$ is of finite index. Further applications will also be presented in the talk.

**The probability of generating a monolithic group**

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Let $L$ be a finite group with a unique minimal normal subgroup, say $N$. We will present some bounds on the conditional probability $P_{L,N}(d)$ that $d$ randomly chosen elements of $L$ generate $L$ given that they generated $L$ modulo $N$. In particular, if $d \geq d(G)$ then $P_{L,N}(d) \geq 1/2$. Several applications to general questions on the generation of finite and profinite groups are described.

**Constructing group extensions with special properties**

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Let $G$ be a finite group acting on a finite abelian group $A$. In this talk I shall present effective methods to determine up to isomorphism all extensions $E$ of $A$ by $G$ such that $A$ is the last non-trivial term of the lower central series respectively the derived series of $E$.

**Classification of embeddings of abelian extensions of $D_n$ into $E_{n+1}$**

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An abelian extension of the special orthogonal Lie algebra $D_n$ is a non-semisimple Lie algebra $D_n \Subset V$, where $V$ is a finite-dimensional representation of $D_n$, with the understanding that $[V, V] = 0$. We determine all abelian extensions of $D_n$ that may be embedded into the exceptional Lie algebra $E_{n+1}$, $n = 5, 6$, and 7. We then classify these embeddings, up to inner automorphism. As an application, we also consider the restrictions of irreducible representations of $E_{n+1}$ to $D_n \Subset V$, and discuss which of these restrictions are or are not indecomposable.

**A Model of Computer Memory**

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Work of Burckel et al. over the past fifteen years or so, has attempted to model computer memory in terms of a certain transformation semigroup of a linear code. Restricting attention to permutations of these codes and to linear transformations of these codes raises natural questions over what groups can be obtained in this way.

**The influence of p-regular class sizes on normal subgroups**

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Let $p$ be a prime number. Let $G$ be a finite group and $N$ be a normal subgroup of $G$. We present some recent results which put forward a strong relation between the conjugacy class sizes in $G$ of the $p$-regular elements of $N$ and the nilpotency of the $p$-complements of $N$.

**On Elementary Free Groups and Some Consequences of the Solution to the Tarski Problems**

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From the positive solution to the Tarski problems by Kharlampovich and Myasnikov and independently Sela it follows that every first order theorem in a nonabelian free group is true in every elementary free group. An elementary free group is a group that shares the first order theory of the class of nonabelian free groups. The class of elementary free groups extends beyond the class of free groups and in particular includes the orientable surface groups of genus $g \geq 2$ and the nonorientable surface groups of genus $g \geq 4$. A well-know theorem of Magnus concerns the normal closures of elements in free groups. A version of this theorem for surface group was proved directly by J. Howie and independently by O. Bogopolski in a quite difficult manner. We show that Magnu's theorem can be given as a sequence of first order sentences and hence is true in an elementary free group and in particular is surface groups of appropriate genus.

This type of result opens up several different types of questions. The first is which additional nontrivial free group results are true in surface groups but difficult to obtain directly. Secondly what first order properties of nonabelian free groups are true beyond the class of elementary free groups.

In this talk we survey a large collection of results on elementary free groups. We show that such groups have cyclic centralizers, are stably hyperolic, satisfy Turner's theorem concerning test elements and are subgroup separable. Further they have tame automorphism groups and constructibe faithful representations in $PSL(2, \mathbb{C})$.

In regard to this second question we consider groups satisfying certain quadratic properties that we call **Lyndon properties** and show that the class of groups satisfying these are closed under many amalgam constructions. Results of Gaglione and Spellman and independently Remeslennikov tie together finitely generated fully residually free groups, commuttaive transitivity and universally free groups. We consider classes of groups that extend this theorem. Work by Ciobanu, Fine and Rosenberger show that these types of classes of groups, that we denote $B\mathcal{X}$, are fairly extensive. Finally we introduce a class of groups defined in terms of conjugacy pinched constructions that generalize both fully residually free groups and groups acting freely on $\mathbb{Z}^n$-trees.

**Algorithms for arithmetic groups with the congruence subgroup property**

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We discuss practical algorithms to compute with arithmetic subgroups $G$ of $SL(n, \mathbb{Q})$, $n > 2$, containing a principal congruence subgroup of known level $m$. Various problems are solved: testing membership in $G$, determining the subnormal structure of $G$, and the orbit problem for $G$. Our approach depends on computing with subgroups of $GL(n, Z/mZ)$.

**Growth in Baumslag-Solitar groups: asymptotics**

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Using the specific example of $BS(2,4)$ we estimate the asymptotics of the spherical growth series. The techniques utilize standard tools from number theory and imply that the actual computation of growth series terms may be an intractable problem.

**Turner's Theorem is not First Order**

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A theorem of E.C. Turner states that in a finitely generated free group the test elements are precisely the elements not contained in any proper retract. Here we show that this theorem is not first order expressible.

**Covering permutation groups**

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In this talk I will present a recent result obtained in a joint work with A. Lucchini. We proved that any noncyclic subgroup $G$ of the symmetric group of degree $n$ is the union of at most $(n+2)/2$ conjugacy classes of proper subgroups of $G$.

**Subgroup structure of branch groups**

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Since their first appearance in the 1980's, branch groups have received much attention due to their unusual properties. Most notably, they serve to answer long-standing group theoretic questions such as the Burnside, Day and Milnor problems. We present results on the subgroup structure of branch groups, focusing on normal subgroups of subgroups of finite index. These will allow us to answer questions concerning commensurability of finitely generated subgroups of branch groups.

**Tensor decomposition, Jordan canonical forms, and ClebschGordan coefficients**

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One motivation for our talk comes from representation theory: decomposing a tensor product of irreducible (or indecomposable) representations as a sum of smaller degree irreducible (or indecomposable) representations. Other motivations come from quantum mechanics and Frobenius algebras.

Consider an $r \times r$ matrix $K_r$ over a field $F$ with 1s on the main diagonal and first upper diagonal (positions $(i,i)$ and $(i,i+1)$) and zeros elsewhere. The tensor product $K_r \otimes K_s$ is a unipotent matrix whose Jordan canonical form is determined by some partition of $rs$. We will describe this partition when the characteristic $p$ of $F$ is small (i.e. $p < r+s-1$). The large characteristic case ($p \geq r+s-1$) was solved recently by Iima and Iwamatsu. This talk will be accessible to postgraduate students.

**On quadratic and cubic action of a rank one group**

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A group $G$ is called an *rank one group with unipotent subgroups* $A$ and $B$ if $G$ is generated by two nilpotent subgroups $A$ and $B$ such that for all $a \in A^*$ there is an element $b(a) \in B^*$ such that $B^a = A^{b(a)}$ and vice versa. A rank one group $G$ is said to act *quadratically* (resp. *cubically*) on a module $V$ if $[V,A,A] = 0$ but $[V,G,G] \neq 0$ (resp. $[V,A,A,A] = 0$ but $[V,A,A] \neq 0$ and $[V,G,G,G] \neq 0$). F. Timmesfeld showed that if a rank one group $G$ acts quadratically on a module $V$, then there is a special quadratic Jordan division algebra such that $G \cong SL_2(J)$. Using Timmesfeld's result, one can prove if $G$ acts cubically on $V$, then some additional conditions imply that $V$ is a pseudoquadratic space of Witt index 1 and $G$ is the isometry group of $V$.

**Relation lifting and the relation gap problem**

Jens Harlander
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Let $F/R$ be a presentation of a group $G$, and let $R/[R,R]$ be the associated relation module. The *relation lifting problem* asks the question: given generators $r_1[R,R], ..., r_n[R,R]$ of the relation module $R/[R,R]$, do there exist *lifts* $r_1c_1, ..., r_nc_n \in R$, where $c_i \in [R,R]$, that normally generate the relation group $R$ ? The relation lifting problem arose first in the work of Wall on finiteness obstructions in 1965. Dunwoody showed in 1972 that relations can not always be lifted. His construction relied on the existence of non-trivial units in the group ring of $\mathbb{Z}_5$. Bestvina and Brady exhibited a presentation of a finitely generated torsion-free group that is not finitely presented but admits a finitely generated relation module. Thus, relation lifting fails in a very strong sense. A difference between the minimal number of relators and the minimal number of relation module generators is called the *relation gap*. Relation gaps in finite presentations have not been found so far, although there is no lack of examples where such a gap is expected to occur. In my talk I intend to survey aspects of the relation lifting and relation gap problem and present some new results.

**Permutation Statistics in Classical Weyl groups**

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There are several results counting involutions (or involutions with no fixed points) with a given number of inversions in the symmetric group. The number of inversions corresponds nicely to the idea of the length of elements, when we view the symmetric group as a Coxeter group of type $A$, which of course raises the question of what happens in types $B$ and $D$. There has been some work and several conjectures in this area.

In this short talk I will present joint work with Peter Rowley, which addresses one of these conjectures.

**Schur indices in GAP**

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The GAP package `wedderga` has provided a function for computing the Wedderburn decomposition of the group algebra of finite group over an abelian number field since 2006. The cyclotomic algebras that appear in its output may not represent division algebras, however, and so in many cases one does not obtain the complete Wedderburn decomposition of the group algebra that one might desire.

I have recently developed new local and global Schur index functions that identify the division algebra part of a cyclotomic algebra in GAP. These have now enabled `wedderga` to include a complete Wedderburn decomposition function for group algebras of finite groups over abelian number fields. In this talk I will give an overview of GAP's new Schur index and Wedderburn decomposition functions.

**Some inverse problems in Baumslag-Solitar groups**

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We investigate inverse problems in Baumslag-Solitar groups

$$BS(1,n) = \langle a, b \mid ab = ba^n \rangle,$$

where $n$ denotes a positive integer. Using known and new results in additive number theory we showed, for example, that if $S$ is a finite subset of the coset $b\langle a\rangle$ in $BS(1,2)$, then $|S^2| = |\{rs \mid r,s \in S\}| \geq 3|S| - 2$. Moreover, if $|S| \geq 3$ and $|S^2| < 4|S| - 4$, then $S$ is a subset of a geometric progression $\{ba^u, ba^{u+d}, ba^{u+2d}, \ldots, ba^{u+(2|S|-2)d}\}$ of length $2|S| - 3$, where $u$ and $d$ denote certain integers depending upon $S$.

**Subgroup structure in the group of infinite triangular matrices**

Waldemar Holubowski
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We make a review of the results known on subgroups in the group $T(\infty, R)$ of infinitely dimensional triangular matrices over the ring $R$. During the last few years, a significant progress has been done towards the characterization of subgroups in $T(\infty, R)$, including the subgroup $UT(\infty, R)$ of all unitriangular infinite matrices over $R$.

In the talk we discuss the following four topics that were developed within the last decade: free subgroups in $UT(\infty, R)$, the lower and derived series in $T(\infty, R)$ and $UT(\infty, R)$, verbal subgroups of $UT(\infty, R)$ and their width, and subgroups of Vershik - Kerov group.

**Representations Arising from an Action on $D$-neighborhoods of Cayley Graphs**

Justin Hughes
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Given $G$ a finite group and a generating set, one can construct the Cayley Graph. With a set $D$ comprised of nonnegative integers one can construct a $D$-neighborhood complex from the Cayley Graph. This neighborhood complex is a simplicial complex and thus it is natural to form an associated chain complex. The group $G$ acts naturally on the chain complex and this leads to an action on the homology of the chain complex. These group actions give rise to several representations of $G$. This work uses tools from group theory, representation theory and homological algebra to further our understanding of the interplay between generated groups (i.e. a group together with a set of generators), corresponding representations on their associated $D$-neighborhood complexes, and the homology of the $D$-neighborhood complexes.

**Practical Algorithms for Matrix Groups**

Alexander Hulpke
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Recent progress on Matrix Group Recognition has made it possible to start designing and implementing higher level algorithms for matrix groups. I will show how to adapt algorithms originally designed for permutation groups to the case of matrix groups, which existing methods immediately transfer and whiich new problems arise. I will also report on concrete implementations of algorithms for element centralizer, element conjugacy and subgroup normalizer.

**Weak Cayley tables and generalized centralizer rings of finite groups**

Stephen Humphries
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For a finite group $G$ we study certain rings $S_G^{(k)}$ called $k$-S-rings, one for each $k \geq 1$, where $S_G^{(1)}$ is the centralizer ring $Z(\mathbb{C}G)$ of $G$. These rings have the property that $S_G^{(k+1)}$ determines $S_G^{(k)}$ for all $k \geq 1$. We study the relationship of $S_G^{(2)}$ with the weak Cayley table of $G$. We show that $S_G^{(2)}$ and the weak Cayley table together determine the sizes of the derived factors of $G$ (noting that a result of Mattarei shows that $S_G^{(1)} = Z(\mathbb{C}G)$ does not). We also show that $S_G^{(4)}$ determines $G$ for any group $G$ with finite conjugacy classes, thus giving an answer to a question of Brauer. We give a criteria for two groups to have the same 2-S-ring and a result guaranteeing that two groups have the same weak Cayley table. Using these results we find a pair of groups of order 512 that have the same weak Cayley table, are a Brauer pair, and have the same 2-S-ring.

**Neighbour-Transitive Codes in Johnson Graphs**

Mark Ioppolo
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A frequently made assumption in coding theory is that the probability of an error occurring does not depend on the position of the error in the codeword, or on the value of the error. The group theoretic analogue of this condition is known as neighbour-transitivity.

A code is called neighbour-transitive if its automorphism group acts transitively on codewords and code-neighbours. Here we outline recent attempts to classify neighbour-transitive codes in Johnson graphs, with emphasis on the case where the code automorphism group is contained in a binary sympletic group acting 2-transitively on a set of quadratic forms.

**Commuting probability and commutator relations**

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The commuting probability of a finite group $G$ is the probability that a randomly chosen pair of elements of $G$ commute. We show that when this number is greater than $1/4$, all the relations between commutators in $G$ are consequences of some universal ones.

**Group matrices old and new**

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The group matrix $X_G$ of a finite group $G$ arises from an ordering $\{g_i\}_{i=1}^n$ of the elements of $G$ and the assignment of a set of variables $\{x_{g_i}\}_{i=1}^n$. It is an encoding of the group operation under one-sided division, $X_G = \{x_{gh}^{-1}\}$. The original problem addressed by Frobenius in the foundational papers of group representation theory was that of the factorisation of $\Theta_G = \det(X_G)$. $X_G$ has appeared in applications in several contexts (random walks, control theory, wavelets...). Given any representation $\rho$ of $G$ there is associated a group matrix $X_G^\rho = \sum_{g\in G} \rho(g)$. Often in practice it is important to decompose $X_G$ into a block diagonal matrix where the blocks are the $X_G^\rho$ for irreducible $\rho$. I will talk about some old and new results.

(1) Usually the diagonalisation of $X_G$ is performed by a similarity transformation over $\mathbb{C}$. However in 1907 Dickson showed for that any $p$-group $P$ there is an ordering of $P$ and a matrix $T$ such that $TPT^{-1}$is a lower triangular matrix with diagonal entries all equal to $\sum_{g\in P} x_g$. Moreover, one such $T$ is a "Pascal triangle matrix", i.e an upper triangular matrix with Pascal's triangle written sideways for the entries. Questions arise as to how the information in $TPT^{-1}$ can be used in applications.

(2) The Grothendieck ring of a finite group is the ring of virtual representations, but it appears hard to express this in terms of group matrices. However, if ideas coming from superalgebras (first investigated by physicists) are used, such a representation can be obtained. It may be possible that "super group matrices" have a meaning in physics.

**Groups, Formal Language Theory, and Decidability**

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In this talk I will give a brief overview of some of the interactions between group theory and formal language theory, in particular, I will focus on the word problem for groups and the study of the word problem as a formal language. I will explain how groups can be classified in terms of the type of automata which accept their word problem. I will then talk about some decidability questions and results in formal language theory on which I have been working which were motivated by the study of the word problem for groups as a formal language. I will not assume any prior knowledge of automata/language theory.

**Public Key Exchange Using Semidirect Product of Groups**

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In this talk I describe a brand new key exchange protocol based on a semidirect product of (semi)groups (more specifically, on extension of a (semi)group by automorphisms), and then focus on practical instances of this general idea. Our protocol can be based on any group, in particular on any non-commutative group. One of its special cases is the standard Diffie-Hellman protocol, which is based on a cyclic group. However, when our protocol is used with a non-commutative (semi)group, it acquires several useful features that make it compare favorably to the Diffie-Hellman protocol. Here we also suggest a particular non-commutative semigroup (of matrices) as the platform and show that security of the relevant protocol is based on a quite different assumption compared to that of the standard Diffie-Hellman protocol.

**Variations on a Theme of I.D. Macdonald**

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In a 1963 paper I.D. Macdonald gave an example of a group in which the cyclic commutator subgroup is not generated by a commutator and he gives sufficient conditions on the group $G$ such that its cyclic commutator subgroup is generated by a commutator.

The question arises, what is the situation for other words in case the associated word subgroup is cyclic, in particular the power word $x^n$, $n$ a positive integer. For $n$ a positive integer, we established sufficient conditions such that $G^n = \langle g^n \mid g \in G\rangle$ is generated by an $n$-th power in case $G^n$ is cyclic and gave examples of groups $G$, where $G^n$ is cyclic but not generated by the $n$-th power of an element.

**Finiteness conjectures in modular representation theory**

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The representation theory of a finite group over a field of positive characteristic $p$ is strongly influenced by the $p$-local structure of the group. The finiteness conjectures of Brauer and Donovan predict that the structure of a $p$-block of a finite group is determined by the order of its defect groups up to finitely many possibilities. These conjectures are still open, but advances in the understanding of the modular representation theory of finite groups of Lie type over the past two decades have led to a spectacular amount of evidence for them. In my talk, I will give an introduction to the conjectures and report on their current status.

**On finite groups with small prime spectrum**

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We survey the recent author's results on finite groups with small prime spectrum. Prime spectrum of a finite group is the set of prime divisors of its order. We describe the chief factors of 3-primary groups and the chief factors of commutator subgroups of 4-primary groups whose prime graphs are disconnected.

As a corollary, 3-primary finite almost simple groups and 4-primary finite simple groups recognizable by prime graph are determined. The complete irreducibility of $GF(2)A_7$-modules in which an element of order 5 acts fixed point freely is proved. Finite groups with the same prime graph as the group $Aut(J_2)$ or $A_{10}$ are described.

**Generators and Relations for a Discrete Subgroup of** $\mathrm{SL}_2(\mathbb{C}) \times \mathrm{SL}_2(\mathbb{C})$

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The main goal is the investigation on the unit group of an order $\mathcal{O}$ in a rational group ring $\mathbb{Q}G$ of a finite group $G$. In particular we are interested in the unit group of $\mathbb{Z}G$. For many finite groups $G$ a specific finite set $B$ of generators of a subgroup of finite index in $\mathcal{U}(\mathbb{Z}G)$ has been given. The only groups $G$ excluded in this result are those for which the Wedderburn decomposition of the rational group algebra $\mathbb{Q}G$ has a simple component that is either a non-commutative division algebra different from a totally definite quaternion algebra or a $2 \times 2$ matrix ring $M_2(D)$, where $D$ is either $\mathbb{Q}$, a quadratic imaginary extension of $\mathbb{Q}$ or a totally definite rational division algebra $\mathcal{H}(a, b, \mathbb{Q})$.

In some of these cases, up to commensurability, the unit group acts discontinuously on a direct poduct of hyperbolic 2- or 3-spaces. The aim is to generalize the theorem of Poincaré on fundamental domains and group presentations to these cases. For the moment we have done this for the Hilbert Modular Group, which acts on $\mathbb{H}^2 \times \mathbb{H}^2$.

**Recent advances on prime graphs of integral group rings.**

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I will report on recent progress in the determination of prime graphs of integral group rings of sporadic simple groups and their automorphism groups, and of groups of order divisible by at most three primes. I will also discuss the reduction of the prime graph question to almost simple groups.

**On some numerical invariants of finite grous**

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All groups considered here are finite. By a numerical invariant of a group $G$ we mean a nonnegative integer, say $I(G)$, which is preserved by isomorphisms. We say, that our invariant $I$ is monotone (on $G$) if $I$ is defined for all subgroups of $G$ and $I(H)$ is less or equal to $I(K)$, whenever $H < K < G$. In this talk I'm going to survey several numerical invariants of finite groups related either to their orders or to generating sets or to lattices of subgroups. Some relations among these invariants will be exhibited. Special attention will be paid to monotonicity of them. In particular, groups with the basis property will be discussed.

**Solvability criteria for finite loops and groups**

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A groupoid $Q$ with a neutral element is called a loop if the equations $ax = ya = b$ have unique solutions $x$ and $y$ for each $a$ and $b$ in $Q$. If we add associativity, the loop is in fact a group. We define two permutation groups associated to the loop, the multiplication group and inner mapping group of the loop. Many properties of loops can be investigated through these groups. We are particularly interested in the solvability of loops. We present the connection between the multiplication groups of loops and connected transversals, formed in 1990. In 1996 Vesanen showed that if the multiplication group of a finite loop is solvable, then the loop is solvable, too. This makes it possible to construct solvability criteria for finite loops in terms of their inner mapping groups, using only group theory.

**Sylow multiplicities in finite groups**

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Let $G$ be a finite group and let $p_1, \ldots, p_m$ be the distinct prime divisors of its order. Let $P = P_1, \ldots, P_m$ be a sequence of Sylow $p_i$-subgroups of $G$. The Sylow multiplicity $m_P(g)$, of an element $g$ of $G$ in the sequence $P$, is the number of distinct factorizations $g = g_1, \ldots, g_m$ for which $g_i$ belongs to $P_i$. I'll review several results and open questions about relations between conditions which are formulated in terms of the numbers $m_P(g)$, and properties of the solvable radical of $G$, the solvable residual of $G$, and ordinary characters of $G$. Part of the talk is based on joint work with Gil Kaplan.

**Images of word maps in almost simple groups and quasisimple groups**

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Let $w$ be a word in the free group of rank $k$. For a group $G$ we can define the word map that sends a $k$-tuple of elements of $G$ to its '$w$-value' by substituting variables and performing all necessary group operations. Let $w(G)$ denote the image of this word map. Clearly, the image of a word map must contain the identity and must be closed under automorphisms of G. Kassabov and Nikolov [1] have constructed words to show that for any simple alternating group there exists a word whose image is precisely the identity and all 3-cycles. More generally, Lubotzky [2] has shown that for any simple group $G$ any automorphism invariant subset which contains the identity is the image of some word map. We will discuss these and other results as well as recent extensions to almost simple groups and quasisimple groups.

[1] M. Kassabov and N. Nikolov. Words with few values in finite simple groups. Q. J. Math., June 2012
[2] Alexander Lubotzky. Images of word maps in finite simple groups. arXiv , (1211.6575v1).

**Width questions for simple groups**

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Let $G$ be a finite group generated by a collection $S$ of subsets of $G$. Define width$(G, S)$ to be the minimal integer $n$ such that $G$ is equal to the union of a product of $n$ subsets in $S$, together with all subproducts. For example, when $S$ consists of a single subset, the width is just the diameter of the Cayley graph of $G$ with respect to this subset. I shall discuss a variety of problems concerning the width of simple groups, mainly in the following cases:

1. the case where $S$ consists of a single subset;
2. the case where $S$ is closed under conjugation.

There are many examples of special interest. For instance, if $S = \{[x, y] : x, y \in G\}$, the set of commutators, the Ore Conjecture asserts that the width is 1 for all finite non-abelian simple groups. The Thompson Conjecture states that there is a conjugacy class $C$ with respect to which the width is 2. There are many recent results and problems concerning the "word width" of simple groups – namely, the width in the case where $S$ consists of all values in $G$ of a fixed word map. There are also combinatorial interpretations of some width problems, such as the estimation of diameters of orbital graphs.

**Rational subsets in groups**

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A subset of a group $G$ is called rational if it is a homomorphic image of a regular set of words. The rational subset membership problem for a finitely generated group $G$ asks whether a given group element belongs to a given rational subset of $G$. This problem generalizes the classical subgroup membership problem (generalized word problem). Only very few classes of groups with decidable rational subset membership problems are known. Examples are free groups, f.g. abelian groups and certain graph groups. In the talk I will give an overview on decidability and undecidability results for the rational subset membership problem. In particular, I will consider wreath products. If time permits, will sketch proofs for the following two results:

1. The rational subset membership problem is decidable for every wreath product $H \wr V$, where H is a finite group and V is virtually free (this includes e.g. the famous lamplighter group).

2. The rational subset membership problem is undecidable for the wreath product $\mathbb{Z} \wr \mathbb{Z}$. Actually, undecidability already holds for a fixed finitely generated submonoid of $\mathbb{Z} \wr \mathbb{Z}$. This implies that Thompson's group $F$ contains a finitely generated submonoid with an undecidable membership problem.

**On Groups with Few Isomorphism Classes of Derived Subgroups**

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Let $G$ be a group. By a derived subgroup in $G$ is meant the commutator subgroup $H'$ of a subgroup $H$ of $G$. We investigate groups which have at most $n$ isomorphism classes of derived subgroups, for a positive integer $n$ ($D_n$-groups). We report some general results on some classes of $D_n$-groups. Then we concentrate on $D_2$ and $D_3$-groups.

**Near supplements and complements in solvable groups of finite rank.**

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Assume that $G$ is a solvable group of finite abelian section rank. Let $K$ be a normal subgroup of $G$ such that $G/K$ is $\pi$-minimax for some set of primes $\pi$. We use cohomology to prove that, if $G/K$ is virtually torsion-free, then $G$ has a $\pi$-minimax subgroup $X$ such that $[G : KX]$ is finite. In addition, we determine conditions that guarantee that $X$ may be chosen so that $K \cap X = 1$.

**Large element orders and the characteristic of finite simple symplectic and orthogonal groups.**

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Let $G$ be a finite simple group of Lie type over a finite field, whose characteristic we denote by $ch(G)$. Suppose that $G$ is defined as a subgroup of $GL_n(q)$ generated by the set of matrices $X$. One of the problems of computational group theory is to find $ch(G)$ by $X$ in polynomial time. The Monte-Carlo algorithm [1] for solving this problem is based on the following property of simple groups of Lie type: if $G$ and $H$ are simple groups of Lie type over fields of odd characteristic such that the sets of element orders of $G$ and $H$ have the same three largest elements, then $ch(G) = ch(H)$ [1, Theorem 1.2]. The groups over fields of characteristic 2 were excluded due to the complexity of calculating maximal element orders of symplectic and orthogonal groups over fields of characteristic 2. We prove that this property holds for all symplectic and some orthogonal groups. We also provide explicit formulae for the two maximal element orders of symplectic groups over fields of characteristic 2.

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**On groups with given spectrum**

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Denote by $\omega(G)$ the spectrum of a group $G$, i. e. the set of its element orders. If the spectrum of $G$ is finite, let $\mu(G)$ be the set of maximal with respect to division elements of $\omega(G)$. We prove that periodic groups with $\mu(G) = \{3, 5, 8\}$, $\mu(G) = \{4, 9\}$ and $\mu(G) = \{8, 9\}$ are all locally finite, and give explicit description of the structure of such groups.

## Four classes of verbal subgroups

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We consider four classes of verbal subgroups in a free group $F$ of rank 2, $\{VN\text{−}verbal\} \subseteq \{P\text{−}verbal\} \subseteq \{R\text{−}verbal\} \subseteq \{M\text{−}verbal\}$. The subgroups in each class define specific properties of corresponding varieties. We show that each of these classes forms a sublattice in the lattice of all subgroups in $F$. Two problems are open.

## On the capability of $p$-groups of class two and prime exponent

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A group $G$ is capable if it is the central quotient of some group $K$, $G \cong K/Z(K)$. I will discuss the problem of determining which $p$-groups of class two and prime exponent are capable. The situation for this class is interesting in that there are known necessary and known sufficient conditions; in general, sufficient conditions for capability are difficult to come by. A full characterization is still unknown, but seems to be within reach of current methods. I will cover a recent result that guarantees capability of $G$ in terms only of the sizes of $[G, G]$ and $G/Z(G)$, as well as some related results and conjectures.

## Finite groups with a metacyclic Frobenius group of automorphisms

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Suppose that a finite group $G$ admits a Frobenius group of automorphisms $FH$ with cyclic kernel $F$ and complement $H$ such that the fixed point subgroup $C_G(H)$ of the complement is nilpotent of class $c$. If $C_G(F) = 1$, then by the Khukhro–Makarenko–Shumyatsky theorem $G$ is nilpotent of $(c, |H|)$-bounded class. We will discuss some resent generalizations of this result to the case of nontrivial $C_G(F)$. In particular, we proved that if $|FH|$ and $|G|$ are coprime, then $G$ has a nilpotent characteristic subgroup of index bounded in terms of $c$, $|C_G(F)|$, and $|FH|$ whose nilpotency class is bounded in terms of $c$ and $|H|$ only. A similar result is also obtained for the ‘modular case, where $G$ and $F$ are finite $p$-groups.

**On the influence of subgroups on structure of finite groups**

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A number of authors studied the structure of a finite group $G$ under the assumption that some of its subgroups are well located in $G$.

Let $G$ be a finite group. Recall that subgroups $A$ and $B$ of $G$ permute if $AB = BA$. A subgroup $H$ is said to be an $s$-permutable subgroup of $G$ if $H$ permutes with every Sylow subgroup of $G$.

Yakov Berkovich investigated the following concept: a subgroup $H$ of a group $G$ is called an NR-subgroup (Normal Restriction) if, whenever $K$ is normal in $H$, $K^G \cap H = K$, where $K^G$ is the normal closure of $K$ in $G$.

In this talk we characterize the class of finite solvable groups in which every subnormal subgroup is normal in terms of NR-subgroups. We also give similar characterizations of the classes of finite solvable groups in which every subnormal subgroup is permutable or s-permutable.

**Groups of exponent 12 without elements of order 12**

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The spectrum of a periodic group is the set of its element orders. We discuss some relations between spectrum and local finiteness, and prove that groups of period 12 without elements of order 12 are locally finite.

**Zassenhaus Conjecture for cyclic-by-abelian groups**

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Let $G$ be a finite group, $\mathbb{Z}G$ the integral group ring and $U(\mathbb{Z}G)$ the group of units of $\mathbb{Z}G$. The most famous open question regarding torsion elements of $U(\mathbb{Z}G)$ is the Zassenhaus Conjecture: Let $u$ be a torsion unit in $U(\mathbb{Z}G)$. Then there exists a unit $x$ in the rational group algebra $\mathbb{Q}G$ such that $x^{-1}ux = g$ or $x^{-1}ux = -g$ for some element $g$ in $G$. We present a proof of this conjecture for cyclic-by-abelian groups. This covers almost all known results on the Zassenhaus Conjecture for solvable groups.

**Algebraic groups and completely reducibility**

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Let $G$ be a reductive algebraic group over a field k of positive characteristic. The notion of a completely reducible subgroup of $G$ generalises the notion of a completely reducible representation (which is the special case when $G = GL_n(k)$). I will describe a geometric approach to the theory of complete reducibility, based on ideas of R.W. Richardson, and I will discuss some recent work involving non-algebraically closed fields.

**On the normal structure of a finite group with restrictions on maximal subgroups**

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In this work we discuss some results about finite groups with restrictions on maximal subgroups which were obtained by N. V. Maslova and D. O. Revin.

We use the term "group" while meaning "finite group."

A subgroup $H$ in a group G is called a *Hall subgroup* if integers $|H|$ and $|G : H|$ are coprime. We say that $G$ is a group with Hall maximal subgroups if each maximal subgroup in $G$ is a Hall subgroup.

We proved, a group $G$ with Hall maximal subgroups contains at most one non-abelian composition factor, the solvable radical $S(G)$ possesses a Sylow normal chain. Furthermore, $G$ acts irreducibly on factors of this chain and factor-group $G/S(G)$ is either trivial or isomorphic to one of the following groups: $PSL_2(7)$, $PSL_2(11)$ or $PSL_5(2)$. In particular all non-abelian composition factors of finite groups with Hall maximal subgroups were described, thus the Problem 17.92 from "Kourovka notebook" [2] is solved.

A group $G$ is a group with complemented maximal subgroups if for every maximal subgroup $M$ of $G$ there exists a subgroup $H$ such that $MH = G$ and $M \cap H = 1$. Using the description of finite groups with Hall maximal subgroups, we proved that in finite groups with Hall all maximal subgroups all maximal subgroups are complemented, thus the conjecture from [1] was proved.

Moreover, it was proved that every finite group with Hall maximal subgroups can be generated by a pair of conjugated elements, thus is was obtained a partial confirmation of P. Shum'jatski's conjecture (see "Kourovka notebook" [2], Problem 17.125).

Let $\pi$ be a set of primes. Given a group G, denote by $\pi(|G|)$ the set of its prime divisors of $|G|$. A value $\pi(G)$ is the *prime spectrum* of a group $G$. A finite group $G$ is *prime spectrum minimal* if $\pi(H) \neq \pi(G)$ for every proper subgroup $H$ of $G$. We have researched some non-abelian composition factors of a prime spectrum minimal group.

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**Embeddings into Thompson's group V and co-CF groups**

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Co-context-free groups provide a natural generalization of context-free groups (which are precisely virtually free groups by work of Muller and Schupp). We give a brief overview of Thompson's group V and co-CF groups and produce new examples of co-CF groups as subgroups of V. Our results and a conjecture by Lehnert would then make V another candidate for a universal co-CF group.

**Finite 3-groups as viewed from class field theory**

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Theorems of Artin and Shafarevich can be used to translate questions about class field towers into questions about groups. In particular questions about 3-class field towers translate into questions about finite 3-groups. In this talk some of these questions will be discussed using p-group generation starting from the elementary abelian group with order 9 and sieving the output using consequences of the theorems of Artin and Shafarevich. From this it can be shown that there are complex quadratic fields whose 3-class field tower has exactly three stages.

**Sidki's Conjecture; showing finiteness of group presentations using amalgams**

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The following:

$$\langle a_1, \dots, a_m | a_i^3 = 1,\ \forall i, (a_i a_j)^2 = 1, i \neq j \rangle$$

is a well-known presentation for the alternating group $A_{m+2}$. In 1982, Sidki asked the question: what if we allow the generators $a_i$ to have order $n$ rather than just order 3? He conjectured that the group $y(m, n)$ given by this presentation was finite. Some results in small cases are known and the conjecture appears to hold (with an interesting pattern of groups occurring), but the question is still open.

We tackle Sidki's problem in a new way, using geometries and group amalgams. This method should not only give an answer to the question of finiteness, but also identify $y(m, n)$. We will describe the background behind the problem, give a brief crash course in geometries and amalgams and show how they apply to this problem.

**Centralizer-like subgroups associated with words in two variables**

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Let $w$ be a word in two variables and let $G$ be a group. In [4], for every element $g$ in $G$ the subsets $W_L^w(g) = \{a \in G \mid w(g, a) = 1\}$ and $W_R^w(g) = \{a \in G \mid w(a, g) = 1\}$ have been considered.

In this talk, based on [2], we examine some sufficient conditions on the group $G$ ensuring that the sets $W_L^w(g)$ and $W_R^w(g)$ are subgroups of $G$ for all $g$ in $G$. In particular, we investigate whether the sets $W_L^w(g)$ and $W_R^w(g)$ are subgroups for words of the form $w(x, y) = C_n[y, x]$, where $C_n$ is a left-normed commutator of weight $n \geq 3$ with entries from the set consisting of $x, y$ and their inverses.

N.D. Gupta [1] considered a number of group laws of the form $C_n = [x, y]$, observing that any finite or solvable group satisfying such a law is abelian. In [3] it has been shown that for $n = 3$, all laws $C_n = [x, y]$ are equivalent to the commutative law, and in [5] the same was established for $n = 4$.

We more specifically investigate the words of the form $w(x, y) = C_3[y, x]$.

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**Simplicity result for groups acting on trees**

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In 1970 Tits studied "property P" for group actions on trees. Property P is an "independence property" that can be interpreted (roughly) as saying that the group acts on one part of the tree independently of how it acts on other parts. Tits showed that if $G$ is a group acting on a tree with property P (and satisfies some non-triviality conditions) then the subgroup generated by the stabilizers of edges is simple. In this talk I want to describe a different independence property and a related simplicity result. This can then be used in the study of automorphism groups of locally finite graphs with more than one end.

**The essential rank of the alternating group.**

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Conjugation families for finite groups were introduced by Alperin in 1967, with his Fusion Theorem. This theorem has been refined by Goldschmidt in 1970 and is now applied to fusion systems, which are used to study the $p$-local structure of a group and modular representation theory. In this talk I will introduce the essential rank of a fusion system for a group, which has been studied for classical groups and the symmetric group, and show how to extend these results to the alternating group.

**On the homology of hyperbolic groups**

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The Rips complex is a locally finite simplicial complex obtained from the Cayley graph of a group. We show how this can be used to construct free resolutions of word-hyperbolic groups that are finitely generated in each dimension. Based on these resolutions we introduce an algorithm to determine the Euler characteristic of a torsion-free word-hyperbolic group and we prove an upper bound for the virtual chomological dimension of a virtually torsion-free word-hyperbolic group.

**On the Covering Number of Small Symmetric Groups**

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Every group $G$ with a finite non-cyclic homomorphic image is a union of finitely many proper subgroups. The minimal number of subgroups needed to cover $G$ is called the covering number of $G$, denoted by $\sigma(G)$. Tomkinson showed that for a soluble group $G$, $\sigma(G) = p^k + 1$, where $p$ is a prime, and he suggested the investigation of the covering number for families of finite non-soluble groups, in particular - simple ones. For the symmetric groups $S_n$ Maroti showed that $\sigma(S_n) = 2^{n-1}$ if $n$ is odd unless $n = 9$, and $\sigma(S_n) \leq 2^{n-2}$ if $n$ is even. We have determined the exact covering number of $S_n$ for some small values of $n$, and found ranges for others. In particular, we show that $\sigma(S_8) = 64$, $\sigma(S_{10}) = 221$, and $243 \leq \sigma(S_9) \leq 256$.

**Beauville Groups**

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Beauville groups arise from a family of surfaces of interest to geometers known as Beauville surfaces. Their definition can be encoded entirely in group-theoretic language and many of the geometric and topological properties of the Beauville surface from which they arise can be recovered from the group itself. Of interest to group theorists is which groups can occur as Beauville groups. We

will present work of the speaker on both examples and non-examples of families of Beauville groups.

**On the number of conjugacy classes in equa-pattern groups**

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A famous open problem due to Graham Higman asks if the number of conjugacy classes in the group of $n \times n$ unipotent upper triangular matrices over the $q$-element field can be expressed as a polynomial function of $q$ for every fixed $n$. In a joint paper with Zoltán Halasi we considered the generalization of the problem for pattern groups and proved that for some pattern groups of nilpotency class two the number of conjugacy classes is not a polynomial function of $q$. Now we make a further generalization. By an equa-pattern group we mean a subgroup of the group of upper unitriangular matrices where some entries are set to be equal to each other, and some other entries are zero. We show that Marcus du Sautoy's notorious group of order $p^9$ can be represented as an equa-pattern group of $13 \times 13$ matrices, and then we calculate the number of conjugacy classes in this group. (That was obtained independently by Michael Vaughan-Lee). This number is not a polynomial function of $p$, in fact, the formula involves the number of points on the elliptic curve $y^2 = x^3 - x$ over the $p$-element field.

**Geometric actions of classical algebraic groups**

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Let $G$ be a classical algebraic group over an algebraically closed field of characteristic $p \geq 0$, with natural module $V$. Various subgroups $H$ of $G$ can be defined naturally in terms of the geometry of $V$ — $H$ may be the stabiliser of a subspace of $V$, or a direct sum decomposition of $V$, or a non-degenerate form on $V$, for example. Let $H$ be such a subgroup and let $\Omega = G/H$ be the corresponding coset variety. We will discuss new results on the dimension of fixed point spaces $C_\Omega(x)$ for elements $x$ in $G$ of prime order.

**Profinite properties of discrete groups**
Alan Reid
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The central theme of these lectures will be: when are finitely generated residually finite groups determined by their finite quotients. As a specific case of this, a famous open problem from the 1970's asks whether a finitely generated free group is determined by its finite quotients.

The lectures will discuss results and ideas in proofs of some progress on these types of questions as well as related results and questions connected to parafree groups. The methods of proof will require technology from the theory of profinite groups as well as computing $L^2$-betti numbers.

**Recent Results on Generalized Baumslag-Solitar Groups**
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A generalized Baumslag-Solitar group is the fundamental group of a graph of groups with infinite cyclic vertex groups and edge groups. There has been considerable activity recently in this area. We will discuss recent work on GBS-groups, including (co)homological properties, the abelianisation, the Schur multiplier, tree dependent and skew tree dependent graphs, how to compute the centre and maximal cyclic normal subgroup, and the relation between GBS-groups and 3-manifold groups.

**Some designs and binary codes preserved by the simple group** Ru **of Rudvalis**
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The simple group Ru of Rudvalis is one the 26 sporadic simple groups. It has a rank-3 primitive permutation representation of degree 4060 which can be used to construct a strongly regular graph $\Gamma$ with parameters $v = 4060$, $k = 1755$, $\lambda = 730$ and $\mu = 780$ or its complement a strongly regular $\widetilde{\Gamma} = (4060, 2304, 1328, 1280)$ graph. The stabilizer of a vertex $u$ in this representation is a maximal subgroup isomorphic to the Ree group $2_{F_4(2)}$ producing orbits $\{u\}$, $\Delta_1$, $\Delta_2$ of lengths 1, 1755, and 2304 respectively. The regular graphs

$\Gamma$, $\widetilde{\Gamma}, \Gamma^R, \widetilde{\Gamma}^R$, $\Gamma^S$ are constructed from the sets $\Delta_1$, $\Delta_2$, $\{u\} \cup \Delta_1$, $\{u\} \cup \Delta_2$, and $\Delta_1 \cup \Delta_2$, respectively. If $A$ denotes an adjacency matrix for $\Gamma$ then $B = J - I - A$, where $J$ is the all-one and $I$ the identity $4060 \times 4060$ matrix, will be an adjacency matrix for the graph $\widetilde{\Gamma}$ on the same vertices. We examine the neighbourhood designs $CD_{1755}$, $CD_{1756}$, $CD_{2304}$, $CD_{2305}$ and $CD_{4059}$ and corresponding binary codes $C_{1755}$, $C_{1756}$, $C_{2304}$, $C_{2305}$, and $C_{4059}$ defined by the binary row span of $A$, $A + I$, $B$, $B + I$ and $A + B$ respectively. $A + I$ and $B + I$ are adjacency matrices for the graphs $\Gamma^R, \widetilde{\Gamma}^R$ obtained from $\Gamma$ and $\widetilde{\Gamma}$, respectively, by including all loops, and thus referred to as reflexive graphs.

**The Asphericity of Injective Labeled Oriented Trees**

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The talk is about the Whitehead conjecture, which states that a subcomplex of an aspherical 2-complex is aspherical. A finite presentation

$$P = \langle x_1, ..., x_n | R_1, ..., R_m \rangle$$

where each relator is of the form $x_i x_k = x_k x_j$ is called a labeled oriented graph (LOG) because we can associate an oriented graph to it in the following way: For each generator $x_i$ there is a vertex $i$ and for each relator $x_i x_k = x_k x_j$ there is an oriented edge from $i$ to $j$ labeled by $k$. A labeled oriented tree (LOT) is a labeled oriented graph where the underlying graph is a tree. LOTs play a central role in the work on the Whitehead conjecture. Results of Howie imply that the finite case of the Whitehead conjecture reduces, up to the Andrews-Curtis conjecture, to the statement that LOT presentations are aspherical (We call a presentation P aspherical if the standard-2-complex modeled over P is aspherical). A labeled oriented tree is called injective, if each generator occurs at most once as an edge label. We show here:

Theorem: Injective labeled oriented trees are aspherical.

**On the exponent of the Schur multiplier**

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We present several bounds for the exponent of the Schur multiplier of a finite group. Some bounds are related to the multipliers of a normal subgroup and its quotient, others are related to the derived length of a $p$-group. These results arise from a wise choice of the representative cocycles, depending if the section is well behaved with the exp $G$-power, or if it is well behaved with conjugation. We discuss the relation of the bounds obtained with the previously known.

**On a theorem of Tate**

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A classical result of Tate states that a finite group is $p$-nilpotent if and only if the restriction map in cohomology from $G$ to a Sylow $p$-subgroup is an isomorphism in dimension 1. In this talk we will discuss how this result can be extended to $p$-solvable groups and, in general to finite groups. As a consequence we will show that the cohomology of normal subgroups in finite groups reads the $p$-solvability. As a byproduct we will show that the $p$-length in a $p$-solvable group and the number of nonabelian chief factors of order divisible by $p$ is bounded by the minimal number of generators of a Sylow $p$-subgroup.

**2-groups with a fixed number of real conjugacy classes**

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We study the finite 2-groups with a fixed number of real conjugacy classes. The order of such groups can be arbitrarily large but we show that it can be bounded if the orders of the elements in a generating set are also fixed. If the number $k$ of real classes is odd we show that the group order can be bounded in terms of $k$ and the nilpotency class although we conjecture that a bound in terms only of $k$ exists. We confirm this conjecture when $k = 7$.

**Real character degrees**
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In the last few years, several results on the set of the degrees of the real valued irreducible characters have been obtained. Ideally, one would like to relate the character degrees with the structure of the group.

Several classical results on the characters degrees of finite groups admit real version. We present some of these theorems.

**Submanifold Projection for $Out(F_n)$**
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One of the most useful tools for studying the geometry of the mapping class group has been the subsurface projections of Masur and Minsky. We propose an analogue for the study of the geometry of $Out(F_n)$ called submanifold projection. We use the doubled handlebody $M_n = \#^n S^2 \times S^1$ as a geometric model of $F_n$, and consider essential embedded 2-spheres in $M_n$, isotopy classes of which can be identified with free splittings of the free group. We interpret submanifold projection in the context of the sphere complex (also known as the splitting complex). We prove that submanifold projection satisfies a number of desirable properties, including a Behrstock inequality and a Bounded Geodesic Image theorem.

**On an inertia factor group of $O^+_{10}(2)$**
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The group $\overline{G} = 2^8{:}O_8^+(2)$ is a group of order 44590694400 and also a maximal subgroup of index 527 of $O^+_{10}(2)$. In turn $2^{10+16}{\cdot}O^+_{10}(2)$ is a maximal subgroup of the monster $M = F_1$. The group $\overline{G}$ has three inertia factor groups namely, $O_8^+(2)$, $SP(6,2)$, $2^6{:}A_8$ and each is of index 1, 120, and 135 respectively in $O_8^+(2)$. The aim of this paper is to compute the Fischer Clifford matrices of $\overline{G}$, which together with its partial character tables are used to compute the full character table of $\overline{G}$. There are 53 Clifford matrices with sizes between 1 and 6. We also give the abstract specification of the group.

**Counting cyclic indentities in specific finite groups**

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Let $Y$ be a $n$-cycle with the set of edges $E = \{e_1, e_2, ..., e_n\}$, and let $G$ be a finite group. In this talk we consider $l_n(G)$ which is the number of functions $f : E \mapsto G$, such $f(e_1)*f(e_2)*\ldots*f(e_n) = f(e_n)*f(e_n-1)*\ldots*f(e_1) = 1$, where 1 is the identity element in $G$, which is a generalization of balanced labeling of edges of a graph to non-abelian groups in the case where the graph is a cycle. We show for every finite group $G$ that $l_3(G) = |G| * |Class(G)|$, and $l_4(G) = |G|^2 * |Class(G)|$, where $|Class(G)|$ is the number of conjugacy classes of the group $G$. We also find formulas of $l_{2n+1}(G)$ and of $l_{2n}(G)$for every metabelian group with a normal abelian subgroup $N$, such that $|c_G(g)| = |G|/|G'|$ for every $g$ not in $N$. (For example $G$ is a finite dihedral group).

**A classification of primitive permutation groups with finite stabilizers**

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In this talk I'll classify all infinite primitive permutation groups possessing a finite point stabilizer, thus extending the seminal O'Nan–Scott Theorem to all primitive permutation groups with finite point stabilizers.

**A Metabelian Group Admitting Integral Polynomial Exponents**

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A classical result of W. Magnus has as a special case a faithful matrix representation of a free metabelian group. We enlarge this group to a group of matrices allowing integral polynomial exponents. We outline a proof that the substitutions of integers for the variable induces a discriminating family of retractions onto the original group. The argument is tricky as we must deal with indeterminate forms 0/0.

**New examples of partial difference sets in finite nonabelian groups**

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A partial difference set (PDS) $S$ in a finite group $G$ is a set of elements of $G$ such that each nonidentity element $g$ of $G$ can be written as a product $ab^{-1}$, where $a, b \in S$, in either $\lambda$ or $\mu$ different ways, depending on whether or not $g$ is in $S$. Whenever $S = S^{-1}$ and $1 \notin S$, the Cayley graph $\mathrm{Cay}(G, S)$ is strongly regular. Very few examples of PDSs are known, and there are especially few known in nonabelian groups. In this talk, a new partial difference set $S$ such that $S = S^{-1}$ and $1 \notin S$ is constructed for each extraspecial group of order $p^3$ and exponent $p^2$, where $p$ is an odd prime, and a new general approach to finding these sets is described.

**$G$-irreducible subgroups of the exceptional algebraic groups**

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A subgroup of an algebraic group $G$ is defined to be $G$-irreducible if it is not contained in any parabolic subgroup of $G$. This is a generalisation of the usual irreducible subgroup of $GL(n, k)$. There is an easy Lemma to find all $G$-irreducible subgroups when $G$ is classical ($A_n$, $B_n$, $C_n$ or $D_n$). We discuss how to use this and other results to find all such subgroups of the exceptional algebraic groups ($G_2$, $F_4$, $E_6$, $E_7$ or $E_8$), in all characteristics, up to conjugacy.

**FA-presentable groups and semigroups**

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The notion of FA-presentability is motivated by an interest in possible approaches for understanding computability in structures. A natural definition would be to take some general model of computation such as a Turing machine; a structure would then be said to be computable if its domain could represented by a set which can be recognized by a Turing machine and if there were decision-making Turing machines for each of its relations. Notwithstanding this, there have been various ideas put forward to restrict the model of computation used; whilst the range of possible structures would decrease, the computation could become more efficient and certain properties of the structures might become decidable.

One interesting approach was introduced by Khoussainov and Nerode who considered structures whose domain and relations can be checked by finite automata; such a structure is said to be FA-presentable. This was inspired, in part, by the theory of automatic groups introduced by Epstein et al; however, the definitions are somewhat different.

We will survey some of what is known about FA-presentable structures, contrasting it with the theory of automatic groups and posing some open questions. We will focus on what is known about FA-presentable groups and semigroups.

**When the commutation of two words gives abelianity**

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N.D. Gupta has proved that groups which satisfy laws $[x, y] \equiv [x,_n y]$ for $n = 2, 3$ are abelian and he has posed the question whether this theorem can be satisfied for every $n$.

Every law $[x, y] \equiv [x,_n y]$ can be written in the form $ab \equiv ba$ where $a, b$ belong to a free group $F_2$ of rank two, and the normal closure of $< a, b >$ coincides with $F_2$. Thus we get the following hypothesis for two words $a, b$ in a free group $F_2$ of rank 2:

*A law $ab \equiv ba$ is equivalent to the abelian law if and only if the normal closure of $< a, b >$ equals $F_2$.*

This hypothesis is an open question. In this talk we discuss some results and problems which appeared during the verification of this hypothesis.

**Prime graphs of finite groups**

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Let $G$ be a finite group. The prime graph of $G$ is a graph whose vertex set is the set of primes dividing some complex irreducible character degree of $G$ and there is an edge between two distinct primes $u$ and $v$ if and only if the product $uv$ divides some character degree of $G$. This graph has been studied extensively over the last 25 years. One of the main questions in this area is to determine which finite simple graphs could be the prime graph of finite groups. In this talk, I will present some results concerning the groups whose prime graphs have no triangles.

**Groups with all subgroups subnormal or soluble of bounded derived length**

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A well-known result, due to W. Mohres [3], states that a group with all subgroups subnormal is soluble, while a result proved, separately, by C. Casolo [1] and H. Smith [4] shows that such a group is nilpotent if it is also torsion-free. Later, Smith generalized these results to groups in which every subgroup is either subnormal or nilpotent. More precisely, he proved, in [6], that a locally (soluble-by-finite) group with all subgroups subnormal or nilpotent is soluble, and the same holds for a locally graded group whose non-nilpotent subgroups are subnormal of bounded defect. Also, in both cases, the nilpotency follows if the group is torsion-free [5].

The purpose of this talk is to discuss locally graded groups with all subgroups subnormal or soluble of bounded derived length [2].

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**Symplectic Alternating Algebras**

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Let $F$ be a field. A symplectic alternating algebra over $F$ consists of a symplectic vector space $V$ over $F$ with a non-degenerate alternating form that is also equipped with a binary alternating product $\cdot$ such that the law $(u\cdot v, w) = (v\cdot w, u)$ holds. These algebraic structures have arisen from the study of 2-Engel groups but seem also to be of interest in their own right with many beautiful properties. We will give an overview with a focus on some recent work on the structure of nilpotent symplectic alternating algebras.

**Certain monomial characters and character correspondences**

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We generalize a theorem of R. Gow on the monomiality of certain characters of solvable groups in two ways. The first generalization allows us to provide a canonical correspondence between certain characters of solvable groups and of certain local subgroups.

**On locally finite groups with bounded centralizer chains**

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The $c$-dimension of a group $G$ is the maximal length of a nested chain of centralizers of subsets in $G$. In 2009 E.I. Khukro proved that a periodic locally soluble group of finite $c$-dimension $k$ is soluble of $k$-bounded derived length. A.V. Borovik conjectured that the number of non-abelian simple composition factors of a locally finite group of finite $c$-dimension $k$ is also $k$-bounded. We prove this conjecture.

**Theorem**. If $G$ is a locally finite group of finite $c$-dimension $k$, then the number of non-abelian simple composition factors of $G$ is less than $5k$.

**An analogue of the Frattini Argument for Hall subgroups**
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The following simple statement is friquently used in the finite group theory.

**Frattini Argument.** *Let $A$ be a normal subgroup of a finite group $G$ and let $S$ be a Sylow $p$-subgroup of $A$ for a prime $p$. Then $G = AN_G(S)$.*

Let $\pi$ be a set of primes. A subgroup $H$ of a group $G$ is called a *$\pi$-Hall subgroup* if every prime divisor of $|H|$ belongs to $\pi$ and $|G : H|$ is not divisible by the elements from $\pi$.

Recall that a group $G$ *satisfies $E_\pi$* if it possesses a $\pi$-Hall subgroup.

It is easy to show that if $A$ is a normal subgroup of a finite group $G$ and $H$ is a $\pi$-Hall subgroup of $G$ then $H \cap A$ is a $\pi$-Hall subgroup of $A$.

The following statement is the main result of the talk:

**Theorem** *Let $\pi$ be a set of primes, $A$ be a normal subgroup of a finite $E_\pi$-group $G$. Then $A$ possesses a $\pi$-Hall subgroup $H$ such that $G = AN_G(H)$.*

We also provide examples showing that the condition $G \in E_\pi$ in Theorem is essential and that the equality $G = AN_G(H)$ does not hold for every $\pi$-Hall subgroup $H$ of $A$.

**$\mathrm{Out}(F_n)$, $\mathrm{GL}(n, \mathbb{Z})$ and everything in between: automorphism groups of right-angled Artin groups**
Karen Vogtmann

A right-angled Artin group (RAAG) is a finitely-generated group which can be presented by saying that some of the generators commute. Free groups and free abelian groups are the extreme examples of RAAGs. Their automorphism groups $\mathrm{GL}(n, \mathbb{Z})$ and $\mathrm{Out}(F_n)$ are complicated and fascinating groups which have been extensively studied. In these lectures I will explain how to use what we know about $\mathrm{GL}(n, \mathbb{Z})$ and $\mathrm{Out}(F_n)$ to study the structure of the (outer) automorphism group of a general RAAG. This will involve both inductive local-to-global methods and the construction of contractible spaces on which these automorphism groups act properly. For the automorphism group of a general RAAG the space we construct is a hybrid of the classical symmetric space on which $\mathrm{GL}(n, \mathbb{Z})$ acts and Outer space with its action of $\mathrm{Out}(F_n)$.

**Zeta functions of groups and rings: recent developments**

Christopher Voll
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Zeta functions of groups are Dirichlet type generating functions which encode group-theoretic data, for example about subgroup or representation growth of infinite groups. Analogously, zeta functions of rings are used in the study of asymptotic properties of rings. A classical example of such a zeta function is the Dedekind zeta function enumerating ideals in the ring of integers of a number field.

Analytic and arithmetic properties of zeta functions of groups and rings often reflect and reveal interesting algebraic properties. In many cases, for instance, zeta functions of groups are Euler products, indexed by the places of a number field. The factors of such Euler products can be studied with a variety of methods, including algebro-geometric ones. Often these local zeta functions have fascinating rationality and symmetry properties.

In my talk I will survey some recent developments in the theory of zeta functions of groups and rings. I will concentrate on general reciprocity results for local zeta functions and new results on representation zeta functions of arithmetic groups.

**Recent Developments in the Study of the Chermak-Delgado Lattice of a Finite Group**

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For a finite group $G$ with subgroup $H$, the Chermak-Delgado measure of $H$ is the product of the order of $H$ with the order of the centralizer of $H$. The set of all subgroups with maximal measure forms a sublattice within the subgroup lattice of $G$, called the Chermak-Delago lattice of $G$. Recently there have been many developments in the study of the Chermak-Delgado lattice, including examples with interesting Chermak-Delgado lattices and theorems for constructing groups with a specific Chermak-Delgado lattice. This talk will discuss both kinds of results and recent developments.

**Tadpole Labelled Oriented Graph Groups and cyclically presented groups**

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A Labelled Oriented Graph group (LOG group) is a group defined by a presentation in which each relation is of the form $x_i x_k = x_k x_j$ (that is, each relation conjugates one generator to another). The presentation can be encoded using a labelled oriented graph. We consider the case when the underlying graph is a tadpole graph and show that the LOG group is the natural HNN extension of a cyclically presented group (which, in certain cases, is a generalized Fibonacci group). We explore properties of the cyclically presented group and the LOG group, and the relationships between the groups.

**Splitting theorems for pro-$p$ groups acting on pro-$p$ trees and 2-generated pro-$p$ subgroups of free pro-$p$ products with procyclic amalgamations**

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We present the result that, under a certain condition, free pro-$p$ products with procyclic amalgamation inherit from its free factors the property of each 2-generated pro-$p$ subgroup being free pro-$p$. This generalizes known pro-$p$ results, as well as some pro-$p$ analogues of classical results in Combinatorial Group Theory. To present our theorem we discuss certain splitting theorems for pro-$p$ groups acting virtually freely on pro-$p$ trees; for instance, any infinite finitely generated pro-$p$ group acting on a pro-$p$ tree such that the restriction of the action to some open subgroup is free splits over an edge stabilizer either as an amalgamated free pro-$p$ product or as a pro-$p$ HNN-extension.

**Intersecting free subgroups in virtually free groups**

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We prove an estimate for the rank of intersection of free subgroups in fundamental groups of finite graphs of groups with finite edge groups. This estimate is analogous to the Hanna Neumann inequality for free groups and the S.V. Ivanov and W.Dicks estimate for free products of groups. An estimate for the rank of intersection of free subgroups in virtually free groups follows from our result as a corollary. We use Bass-Serre theory in the proof.

**Pro-$p$ ends.**

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We shall discuss a pro-$p$ analogue of Stallings' theory of ends.

**On a finite 2,3-generated group of period 12.**

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We use calculations in the computer algebra systems GAP and Magma along with some theoretic results to determine the structure of the largest finite group of period 12 that is generated by an element of order 2 and an element of order 3. In particular, we prove that this group has order $2^{66}.3^7$.

**Hausdorff dimension in pro-$p$ groups**

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Barnea and Shalev showed that a $p$-adic analytic pro-$p$ group has finite Hausdorff dimension spectrum with respect to the $p$th power filtration. In the same work, it was conjectured that the converse holds as well. We prove that the conjecture is true in the solvable case and we also investigate the behaviour of the Hausdorff spectrum in some other cases.