

## FIVE OPEN QUESTIONS

**Question 1.** *Let  $G$  be a finite group and let  $\alpha \in \text{Aut}(G)$ . Find conditions (in terms of the action of  $\alpha$  on  $G$ ) for having  $\alpha \in \Phi(\text{Aut}(G))$ .*

**Question 2.** *Let  $A$  be a finite nontrivial group of odd order. Is it possible to construct a finite group  $X$  such that  $A \leq \text{Aut}(X)$ ,  $1 < F = C_X(A)$  and  $A$  fixes at least two cosets in its natural action on the set  $X/F$  of right cosets with respect to  $F$ ?*

**Question 3.** *Let  $G$  be a group and let  $\phi : G \rightarrow \text{Aut}(G)$  be defined by  $\phi(g) := T_g$ . Then, for all  $g \in G$  and all  $\alpha \in \text{Aut}(G)$  we have*

$$\phi(g^\alpha) = \alpha^{-1} \phi(g) \alpha.$$

*This last equality is also (and obviously) satisfied by the trivial morphism sending every element of  $G$  into the trivial automorphism  $\text{id}_G$ . Find other examples of such "canonic" group morphisms from  $G$  into  $\text{Aut}(G)$ .*

**Question 4.** *Determine the finite groups  $G$  with the property that every subgroup is the fixed point subgroup of some automorphism.*

**Question 5.** *Let  $G$  be a finite group and suppose that  $|\text{Aut}(G)|$  divides  $|G| - 1$ . Is it true that  $\text{Aut}(G)$  acts regularly on  $G$ ?*